A Photoelastic Study of Cracking in Motor Grain Models
A PHOTOELASTIC STUDY OF CRACKING IN MOTOR GRAIN MODELS

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Figure 2: Crosssection of test models.

length of cylinder 376 mm

all dimensions are in mm
• Results from BEM showing "dish" shaped region for normalized SIF for a semi-elliptic surface crack in a pressurized thick walled cylinder.

• $F_1 - K_1 = \text{SIF}; p = \text{pressure}, \Phi = \text{Elliptic Integral of Second Kind.}$

See Table 1 slide.

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Model 3

![Diagram of Model 3 with annotations: MDZ - Material Damage Zone, EVC - Edge View of Crack, Fin Tip, a = 19.6 mm, c = 32.5 mm]

Figure 3: Crack shape and fin tip location for model 3
Model 6b
Center Slice (t = 4.29 mm)

\[
\begin{align*}
P_{sf} & : 2.3 \times 10^{-2} \text{ MPa} \\
c_f & : 175.30 \text{ mm} \\
a_f & : 19.6 \text{ mm} \\
\text{Data zone: } (r_{ave})_2 - (r_{ave})_1 & = 4.2635 - 0.4564 = 3.807 \text{ mm}
\end{align*}
\]

Figure 4: Unmultiplied fringe pattern from a 4.3 mm slice from the center of the crack for Model 6 using a diffused light polariscope for a machined crack denoting location of data zone.
Mode I Algorithm For Determination of Stress Intensity Factor (SIF)

In linear elastic fracture mechanics (LEFM) using the photoelastic approach, one can begin with Mode I near tip equations (Fig. A-1)

\[
\sigma_{ij} = \frac{K_I}{\sqrt{\pi r}} f_i j(\theta) + \sigma_{ij}^0 \quad (i,j = n, z) \tag{A-1}
\]

where \(K_I\) is the Mode I SIF, \(\sigma_{ij}^0\) are the contribution of the nonsingular stresses in the measurement zone, and \(r, \theta\) then are centered at the crack tip. The following expression is computed, in truncated form, along \(\theta = \pi/2\), where fringe spreading is greatest. (Fig. A-2) Thus

\[
r_{\text{max}}^{nz} = \frac{K_{AP}}{\sqrt{8\pi r}} = \frac{K_I}{\sqrt{8\pi r}} + r_0 \tag{A-2}
\]

where \(K_{AP}\) is an "apparent" SIF, which includes the effect of \(\sigma_{ij}^0\) \{i.e., \(r_0 = f(\sigma_{ij}^0)\)\} with the singular effect in the measurement zone. The stress-optic law states that \(r_{\text{max}}^{nz} = \frac{N}{\sqrt{8}} \), where \(N\) is the measured stress fringe order, \(f\) is the material fringe value and \(t\) the slice thickness. Thus \(r_{\text{max}}^{nz}\) is proportional to \(N\) and may be regarded as the measured quantity together with \(r\). By rearranging terms in Eq. A-2 and normalizing, we can obtain

\[
\frac{K_{AP}\Phi}{p\sqrt{\pi a}} = \frac{K_I\Phi}{p\sqrt{\pi a}} + \frac{\sqrt{8}}{p \pi a} \Phi\left(\frac{r}{\sqrt{a}}\right) \tag{A-3}
\]

for a semi-elliptic crack where the coefficient of \(\sqrt{r/a}\) is a constant, \(p\) is the internal pressure and \(a\) is the crack size. \(\Phi\) is an elliptic integral which varies with the aspect ratio of the crack \((a/c)\). Its form is approximated by \(\sqrt{Q}\) where \(Q\) is given in Table I. In general, when applied to cylindrical vessels, the denominator of Eq. A-3 should be \(pR/t\). However, in the present problem geometry, \(R/t = 1\), and the \(R/t\) can be dropped here.

By defining the normalized SIF as

\[
F = \frac{K_{AP}\sqrt{Q}}{p\sqrt{\pi a}} \tag{A-4}
\]

one can plot \(F\) vs. \(\sqrt{r/a}\) and locate the linear zone implied by Eq. A-3, which is the zone dominated by the stress singularity. By extending this line to the origin, the value of \(F\) is determined as shown in Fig. A.3 for Model 1.
Figure 5: Comparison of experimental results for part through cracks in star finned models with same depth cracks in the pressurized cylinder models (GKD) in the central “dish bottom” region.
Figure A-3: Determination of Normalized Stress Intensity Factor (F) from Test Data.
These crack shapes were not semi-elliptical or through the length cracks. Cylinder length was 336mm.

Table 1

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<td>KBF</td>
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Linear dimensions -mm

Pressure = \(N/mm^2\)

\[ F = \frac{K_1\sqrt{Q}}{pR_i/t}\sqrt{\pi a} : \sqrt{Q} = \phi = \text{elliptic Integral of 2nd kind} \]

\[ K \text{ values} = \frac{N}{mm^{3/2}} \]

\[ R_i = \text{Inner cylinder radius} \quad Q \approx 1 + 1.464 \left(\frac{a}{c}\right)^{1.65} \frac{a}{c} < 1 \]

\(\rho_{sf} = \text{Stress freezing pressure} \quad t = \text{Cylinder wall thickness or (distance from fin tip to outer boundary)}\)

subscript notations

KGD = Gouzhong, Kangda and Dongdi (For Cyl.)

BF = Bowie & Freese (for Cyl.)

\[ K_{PSE} = K_{EXP} \left( \frac{K_{BF}}{K_{GKD}} \right) \]

+ These crack shapes were not semi-elliptical or through the length cracks.

* Cylinder length was 336mm

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Summary

By capitalizing on observed similarities between the cracked finned model and a cracked cylinder when placing the fin tip at the inner edge of the cylinder, estimates were made by assuming a plane strain solution for the finned model in finite length models. Based upon the aforementioned limited results, use of a modified plane strain solution appears to yield a slightly conservative prediction for long shallow cracks to significantly conservative prediction for deep part-through cracks.

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